Elasticity and interpretation

Let q_1 and q_2 be the demanded quantities of goods 1 and 2 respectively, and let p_1 and p_2 be their unit prices. If the demand functions that link these prices for a certain population are $q_1 = p_1^{-1.2} p_2^{0.2}$ and $q_2 = p_1^{0.3} p_2^{-0.4}$, the following is requested:

1. Find the direct and cross partial elasticities of both goods. Economically interpret the result.
2. Classify good 1 and good 2.
3. Classify the goods with respect to each other.

Solution

1. We calculate the elasticities:

$$\frac{Eq_1}{Ep_1} = \frac{\partial q_1}{\partial p_1}\frac{p_1}{q_1} = -1.2p_1^{-2.2}p_2^{0.2}\frac{p_1}{p_1^{-1.2}p_2^{0.2}} = \textbf{-1.2}$$

   If the price of good 1 increases by 1%, its demand decreases by approximately 1.2%.

$$\frac{Eq_1}{Ep_2} = \frac{\partial q_1}{\partial p_2}\frac{p_2}{q_1} = 0.2p_1^{-1.2}p_2^{-0.8}\frac{p_2}{p_1^{-1.2}p_2^{0.2}} = \textbf{0.2}$$

   If the price of good 2 increases by 1%, the demand for good 1 increases by approximately 0.2%.

$$\frac{Eq_2}{Ep_2} = \frac{\partial q_2}{\partial p_2}\frac{p_2}{q_2} = -0.4p_1^{0.3}p_2^{-0.8}\frac{p_2}{p_1^{0.3}p_2^{-0.4}} = \textbf{-0.4}$$

   If the price of good 2 increases by 1%, its demand decreases by approximately 0.4%.

$$\frac{Eq_2}{Ep_1} = \frac{\partial q_2}{\partial p_1}\frac{p_1}{q_2} = 0.3p_1^{-0.7}p_2^{-0.4}\frac{p_1}{p_1^{0.3}p_2^{-0.4}} = \textbf{0.3}$$

   If the price of good 1 increases by 1%, the demand for good 2 increases by approximately 0.3%.

2. **They are typical goods since the derivatives of each good with respect to its own price are negative.**

3. **They are substitute goods, as the increase in the price of one good leads to an increase in demand for the other.**